Discounting

Econ 331

Fall 2025

Learning Outcomes/Goals

1. Describe why people value amounts in the future less than the same amounts in earlier periods.
2. Calculate the present value and future value of a given amount using the discount rate in a discrete setting.
3. Explain the relationship between the discount rate and the interest rate.
4. Calculate the present value and future value of a given amount using the discount rate in a continuous setting.

Motivating Example

- Suppose I ask you to choose between receiving \$100 now or \$100 a year from now.

- You would probably choose to receive the \$100 now. Why?

- Well 1 explanation is that you don't want to wait because a year is much further in the future than today. You'd rather have the money now.

- Another explanation is that if I gave you the \$100 now, you could put it into a bank account and earn interest. That \$100 would be worth \$100 plus interest a year from now.

Future Value: Introduction

- Exactly how much would that \$100 be worth in a year?

- Suppose the interest rate is r% and paid annually.
  - This is not compounded continuously, we will come back to that case later.
- In 1 year, that \$100 would be worth the initial \$100 plus the interest earned, or $r \cdot 100$.

- Therefore in 1 year that \$100 would be worth \$$100(1 + r)$.

Future Value: Further Periods

- How much would this \$100 be worth in 2 years?

- This is equivalent to putting our \$100(1+r) back into the bank for another year, earning interest at rate r.

- Our \$100 in two years would be worth $\$100(1+r)(1+r) = 100(1+r)^2$.

- In three years it would be worth $\$100(1+r)(1+r) = 100(1+r)^3$.

Future Values Generally

- Suppose instead of \$100, we want to know the future value (FV) of some amount of money today (\$PV) t periods in the future.
- The interest rate is still r, and interest is added, or accrues, in each period.
  - We can this discrete interest as opposed to continuous interest.
- Let's call this future value FV.
- $FV = PV(1 + r)^t$.

Future Value: Key Equation

$$FV = PV(1 + r)^t$$

- This tells us how much a certain amount of money today (PV) would be worth in the future (FV).
- We can use this same formula to figure out how much a certain amount of money in the future (FV) would be worth today (PV).
- We simply solve the equation above for PV and get

$$PV = \frac{FV}{(1 + r)^t}$$

Where Does r Come From?

- When you lend your money to someone else, you incur a cost. You don't get to use that money.
- You have to wait to receive your money back in some specified amount of time.
- Since this future amount is worth less, r is calculated to fully compensate this impatience.
- In economic models, this actually comes out of the utility function!

Where Does r Come From?

- Suppose based on your preferences, a dollar tomorrow is only worth the same amount as 96 cents today.
- That is to say if someone asked you to pick between a dollar tomorrow or 96 cents tomorrow, you would be indifferent.
- Using these results, see that $\frac{1}{0.96}$ is 1.042, or 1.04.
- We call this the **discount factor**. The **discount rate** is 4%.
- In our models, we set the interest rate equal to the discount rate, so $r = 4\%$.

Continuous Time Interest

- Now suppose instead of using discrete time, interest is compounded continuously.
- From finance, $FV = PVe^{rt}$.
- To find PV, we again solve the above formula for PV.
- We then get $PV = FV \cdot \frac{1}{e^{rt}}$.
- This can also be written as $PV = FV \cdot e^{-rt}$.